

# Online determination of the refractive index of air by ultrasonic speed of sound measurement for interferometric displacement measurements

Virpi Korpelainen and Antti Lassila

Centre for Metrology and Accreditation (MIKES), Lönnrotinkatu 37, P.O. Box 239,  
FIN-00181 Helsinki, Finland  
e-mail: [virpi.korpelainen@mikes.fi](mailto:virpi.korpelainen@mikes.fi)

**Abstract** – An online ultrasonic method for determination of the effective temperature and refractive index of air for interferometric displacement measurements is described and new measurements in demanding environmental conditions are shown. The actual temperature along the laser beam path is difficult to measure with normal temperature sensors, especially in industrial conditions where temperature change may be fast and temperature gradients large. The uncertainty of the position measurement can be significantly reduced especially under demanding conditions.

## 1 INTRODUCTION

The accuracy requirements of interferometric position measurements are increasing in both laboratory and production applications. In laser interferometric position measurements, the environmental conditions affect the results due to their effect on the refractive index of air. For that reason air pressure, temperature and humidity have to be measured accurately. Especially near or inside production equipment temporal and spatial temperature variations may be large, which makes measurement of actual air temperature difficult with traditional methods. For example, with temperature sensors, one can measure temperature only at single points near the actual laser beam path. The response times of the thermometers are also rather long, from several seconds to a few minutes, i.e. fast temperature changes cannot be detected either.

Speed of sound is about two thousand times more sensitive to temperature variations than the refractive index of air and that fact can be used for determination of the effective temperature and refractive index of air [1,2]. The speed of sound can be measured simultaneously with the laser interferometric measurement and over the same path utilising ultrasonic piezo transducers fixed to the optical components of the interferometer. The real-time and real-place refractive index can be determined by using equations for the effective air temperature and refractive index of air as a function of speed of sound ( $f = 50$  kHz), pressure, humidity and CO<sub>2</sub> concentration [1,2].

In this paper, operation under simulated demanding industrial conditions is studied with an updated test set-up. The effect of airflow on acoustic measurement and its compensation by bi-directional measurement as well as operation in warm turbulent air are presented.

## 2 EQUATIONS

Equation for the effective temperature ( $f_t$ ) and refractive index ( $f_n$ ) of air as a function of group velocity of 50 kHz ultrasound ( $c$ ), humidity (mole fraction of water vapour,  $x_w$ ), pressure, ( $p$ ) and CO<sub>2</sub> mole fraction ( $x_w$ ) is [1]

$$f_g = b_0 + b_1c + b_2c^2 + b_3x_w + b_4x_w^2 + b_5x_w^3 + b_6p + b_7p^2 + b_8x_c + b_9x_w c + b_{10}pc + b_{11}x_w p \quad (1)$$

$$g = t \wedge n$$

Coefficients  $b$  are given in table 1. The temperature, humidity, pressure and CO<sub>2</sub> mole fraction ranges for the equations are 19 - 22°C, 0.004 – 0.012 (i.e. 16 - 51%RH), 1000-1040 hPa and 350-450 ppm, respectively. The standard uncertainties of the equations are  $1.7 \times 10^{-8}$  and 15 mK for refractive index and temperature, respectively. The equations are based on experimental speed of sound measurements [1,2] and partly derived from the Cramer equation for speed of sound [3] and the Edlén equations for the refractive index of air [4].

Table 1. Coefficients of equations 1

Refractive index $f_n$			Temperature $f_t$	
$b_0$	1.000682285172		-540.0160	°C
$b_1$	-3.96955965	$\times 10^{-6}$ s/m	1.630328	°Cs/m
$b_2$	5.7733599	$\times 10^{-9}$ s <sup>2</sup> /m <sup>2</sup>	0	°Cs <sup>2</sup> /m <sup>2</sup>
$b_3$	1.364841	$\times 10^{-4}$	86.74	°C
$b_4$	-8.8637	$\times 10^{-4}$	972	°C
$b_5$	2.5538	$\times 10^{-2}$	-27400	°C
$b_6$	7.79906651	$\times 10^{-9}$ 1/Pa	-0.000000111	°C/Pa
$b_7$	1.40365	$\times 10^{-17}$ 1/Pa <sup>2</sup>	0	°C/Pa <sup>2</sup>
$b_8$	9.92	$\times 10^{-6}$	145	°C
$b_9$	-3.58288	$\times 10^{-7}$ s/m	-0.5150	°Cs/m
$b_{10}$	-1.49002013	$\times 10^{-11}$ s/mPa	0	°Cs/mPa
$b_{11}$	3.26201	$\times 10^{-10}$ 1/Pa	0	°C/Pa

## 3 EXPERIMENTAL SET-UP

The experimental setup is shown schematically in figure 1. Two pairs of piezo transducers (APT<sub>x</sub>-y, Marco: ps/mt/13/d) are used with commercial control

boards including a band pass amplifier (BPA) and high voltage amplifier (HVA)) (Marco: ps/mt/2t1/2.1). The transducers are placed symmetrically around a cube corner (CC) and a polarising beam splitter (PBS) (i.e. also symmetrically around the laser beam). The transmitting transducer is selected using a channel scanner. A simultaneous-sampling multifunction data acquisition card (DAC) is used for control and measurement. The sampling rate in the measurements is 5 MHz. The card has an internal clock at 20 MHz with a relative uncertainty of 4 ppm. The length measurement is done using a heterodyne interferometer with a Zeeman stabilised 633 nm He-Ne laser. The ambient humidity and pressure are measured by capacitive sensors and the CO<sub>2</sub> mole fraction is measured by an infrared sensor.

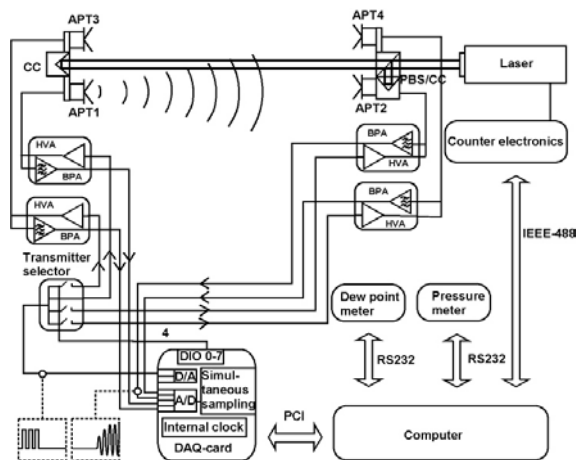


Figure 1. Experimental set-up

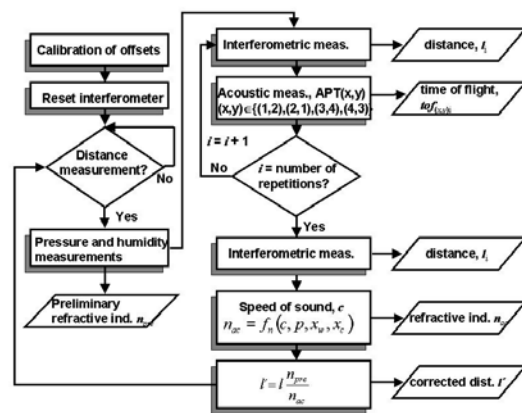


Figure 2. Measurement procedure

For time of flight measurement of ultrasound a burst of three periods of 50 kHz sound is transmitted by each APT in turn and received by the facing APT. Thus, the time of flight is measured twice in both directions (four times totally). The effect of axial airflow is thus eliminated from the average (see section 4.1). The cross-correlation algorithm is used to detect the phase of the received A/D converted signal. The duration of effective temperature measurement is determined by the time of flight multiplied by four (e.g. ~0.06 s for 5 m). Only two detected periods of the signal are used in calculations to reduce the effect of echoes. The mechanical and electrical offsets are defined during the calibration of the system. The measurement procedure is shown as a flow chart in figure 2. The basic measurement algorithm consists of several steps. First, humidity and pressure are measured and the relative humidity is converted to the mole fraction of water. Next, the distance is read from the interferometer, the time of flight of ultrasound is measured, and the distance is measured again. In the measurements presented in this paper, the acoustic measurements were done twice during each measurement. The speed of sound, the effective air

temperature and the refractive index are calculated. Finally, the correction to the measured distance is calculated.

The set-up and measurement procedure is explained in more detail elsewhere [1,2]. The set-up and measurement software is updated to allow higher measurement rate. Depending on the distance one measurement cycle takes ~0.3...1 s including analysis.

## **4 RESULTS AND DISCUSSION**

In the following test measurements, demanding measurement conditions e.g. in workshops are simulated. First, sensitivity of the set-up for axial airflow was studied. Then operation under severe industrial condition was tested by exposing the beam path to turbulent warm airflow.

### **4.1 Effect of airflow**

The speed of sound is relative to the velocity of the media. Therefore, airflow affects the speed of sound. In the measurement shown in figure 3, the acoustic measurement was disturbed by airflow caused by two fans operated in turns. The maximum airflow was ~0.2 m/s and the direction of the airflow was changed a few times. The speeds of sound measured with each acoustic transducer in turn are shown in figure 3. The effect of the airflow can be clearly seen, but the effect could be eliminated by an average of the speeds measured in both directions. Most of the variations in the average speed of sound were caused by small temperature variations (~50 mK), and their effect on the refractive index and interferometrically measured distance could be compensated for (see figure 3).

### **4.2 Warm turbulent air**

In the interferometric distance measurements outside well-controlled laboratories, the measurement of the temperature is often a big problem. The temporal and spatial changes in the temperature can be large and fast. Warm turbulent air causes very fast and large local temperature variations, which are very difficult to compensate for. In the measurement shown in figure 4, a fixed distance was measured using a laser interferometer and the measurement was disturbed by blowing warm air across the beam path. The maximum temperature of the airflow was ~23...25°C and changes in average temperature along the laser beam path were up to 0.8°C. The changes in interferometrically

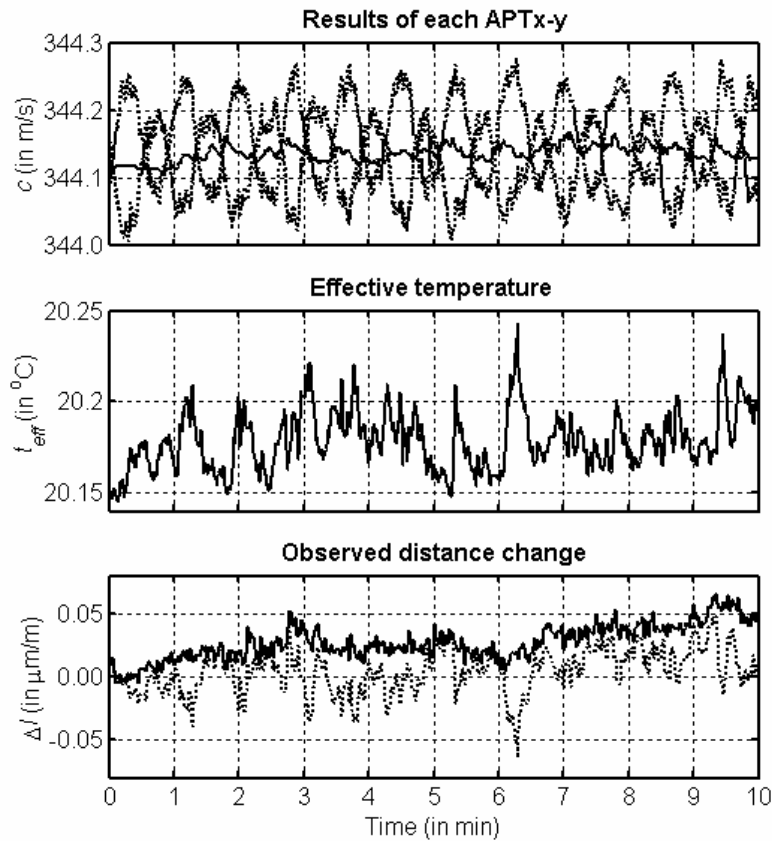


Figure 3. Effect of airflow on acoustic measurement. Speed of sound measured ( $c$ ) with each of the transducers (----) and an average of them (—), changes in effective temperature ( $t_{eff}$ ) and uncorrected (----) and acoustically corrected (—) interferometrically measured distance ( $\Delta l$ ).

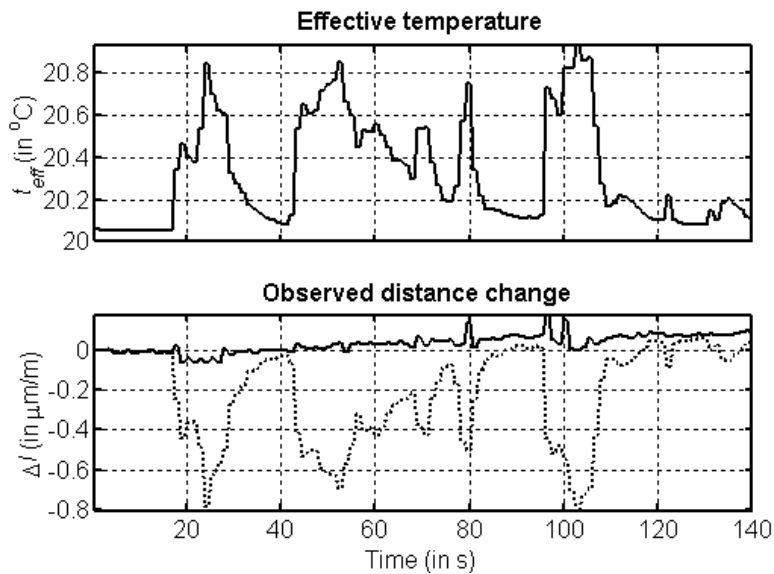


Figure 4. Effect of warm airflow on interferometric measurement. Changes interferometrically measured distance ( $\Delta l$ ) (uncorrected ---- and acoustically corrected —) and in effective temperature ( $t_{eff}$ ). The total distance in the measurement was  $\sim 2.9$  m

measured distance were  $\sim 0.8 \mu\text{m/m}$  in the uncompensated distance, but only  $\sim 0.05 \mu\text{m/m}$  in the acoustically compensated distance. The slight increasing trend in compensated distance was caused by thermal expansion, which could not be completely avoided during the measurement. Small peaks in the compensated distance were caused by imperfect synchronisation of the time of flight and distance measurements.

## 5 CONCLUSIONS

The accuracy of interferometric distance measurements can be improved with online ultrasonic measurement of the refractive index of air. The accuracy can be improved especially in measurements under difficult environmental conditions, e.g. in workshops, where temperature variations may be large and fast. The main advantage of the acoustic method compared to the other commonly used methods for determination of the refractive index of air are that the refractive index can be measured simultaneously with the interferometric measurement and along the laser beam path. The standard uncertainty of  $2.6 \times 10^{-8}$  can be achieved in laboratory conditions, and for example, an effect of warm airflow can be reduced from  $1 \mu\text{m/m}$  to less than  $0.1 \mu\text{m/m}$ . The effect of airflow on ultrasonic measurement can be well compensated for by bi-directional measurement of the speed of sound. The updated test set-up and software allow better and faster online compensation of the refractive index of air.

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